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Your Roll No.....

Sr. No. of Question Paper : 738

B

Unique Paper Code : 32351201

Name of the Paper : BMATH203 – Real Analysis

Name of the Course : B.Sc. (H) Mathematics

Semester : II

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All Questions are Compulsory.
3. Attempt any two parts from each question.
4. All Questions are of equal marks.

1. (a) State the completeness property of $\mathbb{R}$, hence show that every non-empty set of real numbers which is bounded below, has an infimum in $\mathbb{R}$.

(b) Show that if A and B are bounded subsets then $A \cup B$ is a bounded set and $\sup(A \cup B) = \max\{\sup A, \sup B\}$.

(c) State and prove nested interval property.

(d) Define an open set and closed set in $\mathbb{R}$.

Show that if $a, b \in \mathbb{R}$, then the open interval (a, b) is an open set.

Is a closed interval a closed set?

2. (a) Let S be a bounded set in $\mathbb{R}$ and let S_0 be a non-empty subset of S . Show that

$$\inf S \leq \inf S_0 \leq \sup S_0 \leq \sup S$$

(b) State Archimedean property. Hence, prove that

$$S = \left\{ \frac{1}{n}, n \in \mathbb{N} \right\} \text{ then } \inf S = 0.$$

(c) If $S \subseteq \mathbb{R}$ is non-empty. Show that S is bounded if and only if there exists a closed bounded interval I such that $S \subseteq I$.

- (d) If $x, y, z \in \mathbb{R}$ and $x \leq z$. Show that $x \leq y \leq z$ if and only if $|x - y| + |y - z| = |x - z|$. Interpret this geometrically.

3. (a) Prove that a convergent sequence of real numbers is bounded.

Is the converse true? Justify.

- (b) Let (x_n) be a sequence of positive real numbers

such that $\lim_{n \rightarrow \infty} \left(\frac{x_{n+1}}{x_n} \right) = L$ exists. If $L < 1$, then

(x_n) converges and $\lim_{n \rightarrow \infty} (x_n) = 0$.

- (c) Prove that if $C > 0$, then $\lim_{n \rightarrow \infty} (C^{1/n}) = 1$.

- (d) Let $x_1 > 1$ and $x_{n+1} = 2 - \frac{1}{x_n}$ for $n \in \mathbb{N}$. Show

that (x_n) is bounded and monotone. Also find the limit.

4. (a) Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers that converge to x and y respectively. Then the product sequence $X.Y$ converges to $x.y$.

(b) Let $X = (x_n)$ be a bounded sequence of real numbers and let $x \in \mathbb{R}$ have the property that every convergent subsequence of X converges to x . Then the sequence X is convergent to x .

(c) Discuss the convergence of the sequence (x_n) ,

where
$$x_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \text{ for } n \in \mathbb{N}.$$

(d) Use the definition of the limit of the sequence to find the following limits

(i)
$$\lim_{n \rightarrow \infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

(ii)
$$\lim_{n \rightarrow \infty} \left(\frac{3n+1}{2n+5} \right)$$

5. (a) Prove that a necessary condition for the convergence of an infinite series $\sum a_n$ is $\lim_{n \rightarrow \infty} a_n = 0$.

Is the condition sufficient? Justify with the help of an example.

(b) Prove that the geometric series $1 + r + r^2 + \dots$ converges for $0 \leq r \leq 1$ and diverges for $r \geq 1$.

(c) Test for convergence, the following series :

(i) $\frac{1}{5} + \frac{2!}{5^2} + \frac{3!}{5^3} + \dots$

(ii) $\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n-1}}{n}$

(d) Prove that the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$ converges if and only if $-1 \leq x \leq 1$.

6. (a) State and prove Cauchy's n^{th} root test for positive term series.

(b) Prove that the series $1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \dots$ converges for $p > 1$ and diverges for $p \leq 1$.

(c) Test for convergence, the following series :

(i) $\sum_{n=1}^{\infty} [\sqrt[3]{n^3 + 1} - n]$

(ii) $\sum_{n=1}^{\infty} 2^{-n}(-1)^n$

(d) Prove that every absolutely convergent series is

convergent. Show that the series $\sum (-1)^n \frac{n+2}{2^n+5} x^n$

converges for all the real values of x .

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